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Fitting of Normal Distribution by Using Areas Method between Rainfall and Ground Water Levels - A Case Study

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ABSTRACT

Present paper deals with the application of 'Normal distribution' to analyze and predict Rainfall (RF) and Ground water levels (GWLs) in Anantapuramu district based on the data collected from January 2007 to December 2016. With Normal distribution by using areas method, for the purpose of analysis the district is divided into five zones or Revenue Divisions (RD) namely, (1) Anantapuramu RD (2) Penukonda RD (3) Kadiri RD (4) Kalyandurg RD (5) Dharmavaram RD. The values of Normal distribution have been calculated by using areas method and compared among them by using the data and conclusions are drawn based on the results obtained.

Keywords: Rainfall, Ground water level, Normal distribution, Areas method, Prediction.

1. Introduction

I have discussed 'Distribution Theory' for different distributions like. Binomial Distribution-Direct and Recurrence Relation Method, Negative Binomial Distribution Recurrence Relation Method and Poisson distribution direct and recurrence relation methods already I have analyzed; now in this paper, I will fit **Normal distribution by using areas method.**

The data is collected on Average Rainfall and Average Ground Water Levels are given in the following Table-1 for a ready reference [1, 2, 3, 4, 5, 6, 7, 8 and 9].

Table 1. Average Rainfall and Average Ground water levels data from 2007 to 2016

Year	Zone-I		Zone-II		Zone-III		Zone-IV		Zone-V	
	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL	RF (in mm)	GWL
2007	65.60	10.57	58.20	22.58	67.20	14.23	52.00	14.97	60.50	17.03
2008	53.90	9.96	77.90	20.73	65.20	9.27	61.30	10.88	62.70	9.09
2009	45.40	12.17	50.60	17.53	46.30	11.08	57.10	9.58	38.70	10.24
2010	53.90	12.74	71.50	15.02	70.80	12.03	64.60	8.58	56.30	11.79
2011	39.50	12.69	42.30	15.20	48.90	11.48	31.80	8.93	36.60	12.84
2012	43.20	14.98	43.40	20.49	45.30	16.08	40.50	13.76	41.90	13.22
2013	35.00	15.94	52.30	23.03	47.10	18.69	34.80	16.98	38.10	14.30
2014	31.10	15.87	30.30	23.40	27.10	21.16	37.10	18.92	22.80	16.30
2015	44.10	14.90	62.60	26.88	66.30	25.80	46.00	19.26	54.30	17.66
2016	33.50	15.57	33.40	27.27	32.30	15.35	25.70	19.51	30.10	16.15

2. Statistical Analysis

To analyze **Rainfall** and **Ground Water Levels** through **Normal distribution by using areas method** for different zones we can consider given as follows:

The p.d.f. of Normal distribution is given by,

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2}; -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0 \qquad (1)$$

Here, μ and σ are called the parameters of Normal distribution. These are estimated as,

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N=\sum_{i=1}^{n} f_i} = \hat{\mu} \qquad (2)$$

$$\sigma^2 = \frac{1}{N} f_i x_i^2 - (\bar{x})^2 = \hat{\sigma}^2 \qquad (3)$$

Therefore, the p.d.f. of Normal distribution fitted to the given data is,

$$f(x;\hat{\mu},\hat{\sigma}) = \frac{1}{\hat{\sigma}\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-\hat{\mu}}{\hat{\sigma}})^2}; -\infty < x < \infty, -\infty < \hat{\mu} < \infty, \hat{\sigma} > 0 \qquad (4)$$

To find the Expected Frequencies

To find the Expected Frequencies, we use the following steps.

Step-I: We convert the given data into the continuous frequency distribution.

Step-II: The Standard Normal Variate z is defined as,

$$z_i = \frac{(x_i-\mu)}{\sigma} \qquad (5)$$

Where, x_i are the lower limits of the class interval, the areas of each z_i are taken from Normal tables, for the calculation of frequencies.

The fitted Normal distribution by using areas method for Average RF and Average GWLs:

A: For Average Rainfall

Zone-I

The p.d.f. of Normal distribution is given by,

$$f(x;\mu,\sigma) = \frac{1}{2.91\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-4.96}{2.91})^2}; -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

C.I	**Lower Limit**	$\boldsymbol{z_i = \frac{(x_i-\mu)}{\sigma}}$	**Areas $\phi(z_i)$**	$\Delta \times \phi(z_i)$	**N×($\Delta\times\phi(z_i)$) frequencies**
−∞---0.5	−∞	−∞	0.5		
0.5----1.5	0.5	-1.53	0.4370	-0.06	-26.71
1.5----2.5	1.5	-1.19	0.3830	-0.05	-22.26
2.5----3.5	2.5	-0.85	0.3023	-0.08	-35.62

3.5----4.5	3.5	-0.50	0.1915	-0.11	-48.97
4.5----5.5	4.5	-0.16	0.0636	-0.13	-57.88
5.5----6.5	5.5	0.19	0.0759	0.01	4.45
6.5----7.5	6.5	0.53	0.2019	0.13	57.88
7.5----8.5	7.5	0.87	0.3078	0.11	48.97
8.5----9.5	8.5	1.22	0.3888	0.08	35.62
9.5----10.5	9.5	1.56	0.4406	0.05	22.26
10.5--+∞	10.5	1.90	0.4713	0.03	13.36

Zone-II

The p.d.f. of Normal distribution is given by,

$$f(x;\mu,\sigma)=\frac{1}{2.86\sqrt{2\pi}}e^{\frac{-1}{2}(\frac{x-5.03}{2.86})^2};-\infty<x<\infty,-\infty<\mu<\infty,\sigma>0$$

C.I	**Lower Limit**	$z_i=\frac{(x_i-\mu)}{\sigma}$	**Areas ϕ (z_i)**	**Δ × ϕ (z_i)**	**N×(Δ× ϕ (z_i)) frequencies**
−∞---0.5	−∞	−∞	0.5		
0.5----1.5	0.5	-1.58	0.4429	-0.06	-31.35
1.5----2.5	1.5	-1.23	0.3907	-0.05	-26.13
2.5----3.5	2.5	-0.88	0.3106	-0.08	-41.80
3.5----4.5	3.5	-0.53	0.2019	-0.11	-57.48
4.5----5.5	4.5	-0.19	0.0759	-0.13	-67.93
5.5----6.5	5.5	0.16	0.0636	-0.01	-5.23
6.5----7.5	6.5	0.51	0.1950	0.13	67.93
7.5----8.5	7.5	0.86	0.3051	0.11	57.48
8.5----9.5	8.5	1.21	0.3869	0.08	41.80
9.5----10.5	9.5	1.56	0.4406	0.05	26.13
10.5--+∞	10.5	1.91	0.4719	0.03	15.68

Zone-III

The p.d.f. of Normal distribution is given by,

$$f(x;\mu,\sigma)=\frac{1}{2.87\sqrt{2\pi}}e^{\frac{-1}{2}(\frac{x-5.04}{2.87})^2};-\infty<x<\infty,-\infty<\mu<\infty,\sigma>0$$

C.I	**Lower Limit**	$z_i=\frac{(x_i-\mu)}{\sigma}$	**Areas ϕ (z_i)**	**Δ × ϕ (z_i)**	**N×(Δ× ϕ (z_i)) frequencies**
−∞---0.5	−∞	−∞	0.5		

0.5----1.5	0.5	-1.58	0.4429	-0.06	-30.99
1.5----2.5	1.5	-1.23	0.3907	-0.05	-25.83
2.5----3.5	2.5	-0.89	0.3133	-0.08	-41.32
3.5----4.5	3.5	-0.54	0.2054	-0.11	-56.82
4.5----5.5	4.5	-0.19	0.0759	-0.13	-67.15
5.5----6.5	5.5	0.16	0.0636	-0.01	-5.17
6.5----7.5	6.5	0.51	0.1950	0.13	67.15
7.5----8.5	7.5	0.86	0.3051	0.11	56.82
8.5----9.5	8.5	1.21	0.3869	0.08	41.32
9.5----10.5	9.5	1.55	0.4394	0.05	25.83
10.5--+∞	10.5	1.90	0.4713	0.03	15.50

Zone-IV

The p.d.f. of Normal distribution is given by,

$$f(x;\mu,\sigma) = \frac{1}{2.81\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-4.92}{2.81})^2}; -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

C.I	**Lower Limit**	$z_i = \frac{(x_i - \mu)}{\sigma}$	**Areas** $\phi(z_i)$	$\Delta \times \phi(z_i)$	**N×(Δ× ϕ** (z_i)**) frequencies**
−∞---0.5	−∞	−∞	0.5		
0.5----1.5	0.5	-1.57	0.4418	-0.06	-27.05
1.5----2.5	1.5	-1.22	0.3888	-0.05	-22.55
2.5----3.5	2.5	-0.86	0.3051	-0.08	-36.07
3.5----4.5	3.5	-0.51	0.1950	-0.11	-49.60
4.5----5.5	4.5	-0.15	0.0596	-0.14	-63.13
5.5----6.5	5.5	0.21	0.0832	0.02	9.02
6.5----7.5	6.5	0.56	0.2123	0.13	58.62
7.5----8.5	7.5	0.92	0.3212	0.11	49.60
8.5----9.5	8.5	1.27	0.3980	0.08	36.07
9.5----10.5	9.5	1.63	0.4484	0.05	22.55
10.5--+∞	10.5	1.99	0.4767	0.03	13.53

Zone-V

The p.d.f. of Normal distribution is given by,

$$f(x;\mu,\sigma) = \frac{1}{2.92\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-4.98}{2.92})^2}; -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

C.I	Lower Limit	$z_i = \frac{(x_i - \mu)}{\sigma}$	Areas $\phi(z_i)$	$\Delta \times \phi(z_i)$	N×(Δ× $\phi(z_i)$) frequencies
−∞---0.5	−∞	−∞	0.5		
0.5----1.5	0.5	-1.53	0.4370	-0.06	-26.52
1.5----2.5	1.5	-1.19	0.3830	-0.05	-22.10
2.5----3.5	2.5	-0.85	0.3023	-0.08	-35.36
3.5----4.5	3.5	-0.51	0.1950	-0.11	-48.62
4.5----5.5	4.5	-0.16	0.0636	-0.13	-57.46
5.5----6.5	5.5	0.18	0.0714	0.01	4.42
6.5----7.5	6.5	0.52	0.1985	0.13	57.46
7.5----8.5	7.5	0.86	0.3051	0.11	48.62
8.5----9.5	8.5	1.21	0.3869	0.08	35.36
9.5----10.5	9.5	1.55	0.4394	0.05	22.10
10.5--+∞	10.5	1.89	0.4706	0.03	13.26

B: For Average Ground water levels

Zone-I

The p.d.f. of Normal distribution is given by,

$$f(x;\mu,\sigma) = \frac{1}{2.79\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-5.91}{2.79})^2}; -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

C.I	Lower Limit	$z_i = \frac{(x_i - \mu)}{\sigma}$	Areas $\phi(z_i)$	$\Delta \times \phi(z_i)$	N×(Δ× $\phi(z_i)$) frequencies
−∞---0.5	−∞	−∞	0.5		
0.5----1.5	0.5	-1.94	0.4738	-0.03	-4.06
1.5----2.5	1.5	-1.58	0.4429	-0.03	-4.06
2.5----3.5	2.5	-1.22	0.3888	-0.05	-6.77
3.5----4.5	3.5	-0.86	0.3051	-0.08	-10.83
4.5----5.5	4.5	-0.51	0.1950	-0.11	-14.89
5.5----6.5	5.5	-0.15	0.0596	-0.14	-18.95
6.5----7.5	6.5	0.21	0.0832	0.02	2.71
7.5----8.5	7.5	0.57	0.2157	0.13	17.60
8.5----9.5	8.5	0.93	0.3238	0.11	14.89
9.5----10.5	9.5	1.29	0.4015	0.08	10.83
10.5--+∞	10.5	1.65	0.4505	0.05	6.77

Zone-II

The p.d.f. of Normal distribution is given by,

$$f(x;\mu,\sigma)=\frac{1}{3.01\sqrt{2\pi}}e^{\frac{-1}{2}(\frac{x-5.84}{3.01})^2};-\infty<x<\infty,-\infty<\mu<\infty,\sigma>0$$

C.I	Lower Limit	$z_i=\frac{(x_i-\mu)}{\sigma}$	Areas $\phi(z_i)$	$\Delta\times\phi(z_i)$	$N\times(\Delta\times\phi(z_i))$ frequencies
−∞---0.5	−∞	−∞	0.5		
0.5----1.5	0.5	-1.77	0.4616	-0.04	-8.49
1.5----2.5	1.5	-1.44	0.4251	-0.04	-8.49
2.5----3.5	2.5	-1.11	0.3655	-0.06	-12.73
3.5----4.5	3.5	-0.78	0.2823	-0.08	-16.97
4.5----5.5	4.5	-0.45	0.1736	-0.11	-23.33
5.5----6.5	5.5	-0.11	0.0438	-0.13	-27.58
6.5----7.5	6.5	0.22	0.0871	0.04	8.49
7.5----8.5	7.5	0.55	0.2088	0.12	25.46
8.5----9.5	8.5	0.88	0.3106	0.10	21.21
9.5----10.5	9.5	1.22	0.3888	0.08	16.97
10.5--+∞	10.5	1.55	0.4394	0.05	10.61

Zone-III

The p.d.f. of Normal distribution is given by,

$$f(x;\mu,\sigma)=\frac{1}{2.82\sqrt{2\pi}}e^{\frac{-1}{2}(\frac{x-6.15}{2.82})^2};-\infty<x<\infty,-\infty<\mu<\infty,\sigma>0$$

C.I	Lower Limit	$z_i=\frac{(x_i-\mu)}{\sigma}$	Areas $\phi(z_i)$	$\Delta\times\phi(z_i)$	$N\times(\Delta\times\phi(z_i))$ frequencies
−∞---0.5	−∞	−∞	0.5		
0.5----1.5	0.5	-2.00	0.4772	-0.02	-3.10
1.5----2.5	1.5	-1.65	0.4505	-0.03	-4.66
2.5----3.5	2.5	-1.29	0.4015	-0.05	-7.76
3.5----4.5	3.5	-0.94	0.3264	-0.08	-12.41
4.5----5.5	4.5	-0.59	0.2224	-0.10	-15.52
5.5----6.5	5.5	-0.23	0.0910	-0.13	-20.17
6.5----7.5	6.5	0.12	0.0478	-0.04	-6,21
7.5----8.5	7.5	0.48	0.1844	0.14	21.72

8.5----9.5	8.5	0.83	0.2967	0.11	17.07
9.5----10.5	9.5	1.19	0.3830	0.09	13.97
10.5--+∞	10.5	1.54	0.4382	0.06	9.31

Zone-IV

The p.d.f. of Normal distribution is given by,

$$f(x;\mu,\sigma)=\frac{1}{2.98\sqrt{2\pi}}e^{\frac{-1}{2}(\frac{x-6.12}{2.98})^2};-\infty<x<\infty,-\infty<\mu<\infty,\sigma>0$$

C.I	Lower Limit	$z_i=\frac{(x_i-\mu)}{\sigma}$	Areas $\phi(z_i)$	$\Delta\times\phi(z_i)$	N×($\Delta\times\phi(z_i)$) frequencies
−∞---0.5	−∞	−∞	0.5		
0.5----1.5	0.5	-1.89	0.4706	-0.03	-4.24
1.5----2.5	1.5	-1.55	0.4394	-0.03	-4.24
2.5----3.5	2.5	-1.21	0.3869	-0.05	-7.07
3.5----4.5	3.5	-0.88	0.3106	-0.08	-11.31
4.5----5.5	4.5	-0.54	0.2054	-0.11	-15.55
5.5----6.5	5.5	-0.21	0.0832	-0.12	-16.96
6.5----7.5	6.5	0.13	0.0517	-0.03	-4.24
7.5----8.5	7.5	0.46	0.1772	0.13	18.38
8.5----9.5	8.5	0.80	0.2881	0.11	15.55
9.5----10.5	9.5	1.13	0.3708	0.08	11.31
10.5--+∞	10.5	1.47	0.4292	0.06	8.48

Zone-V

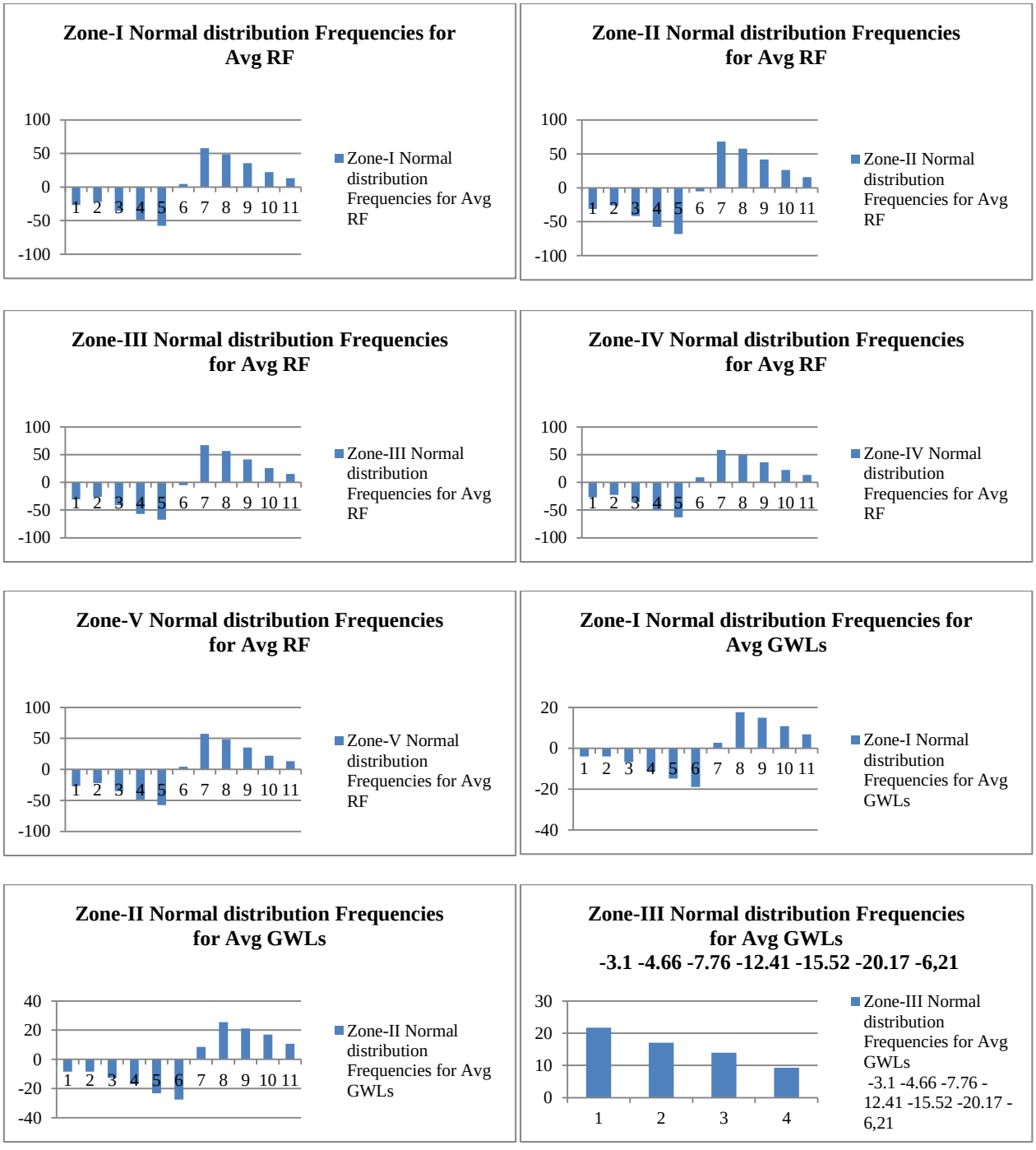
The p.d.f. of Normal distribution is given by,

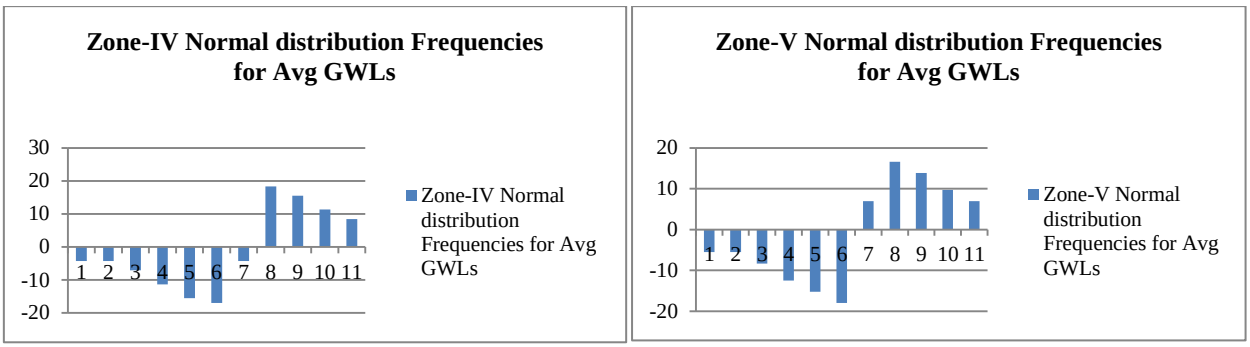
$$f(x;\mu,\sigma)=\frac{1}{2.95\sqrt{2\pi}}e^{\frac{-1}{2}(\frac{x-5.83}{2.95})^2};-\infty<x<\infty,-\infty<\mu<\infty,\sigma>0$$

C.I	Lower Limit	$z_i=\frac{(x_i-\mu)}{\sigma}$	Areas $\phi(z_i)$	$\Delta\times\phi(z_i)$	N×($\Delta\times\phi(z_i)$) frequencies
−∞---0.5	−∞	−∞	0.5		
0.5----1.5	0.5	-1.81	0.4649	-0.04	-5.54
1.5----2.5	1.5	-1.47	0.4292	-0.04	-5.54
2.5----3.5	2.5	-1.13	0.3708	-0.06	-8.32
3.5----4.5	3.5	-0.79	0.2852	-0.09	-12.48
4.5----5.5	4.5	-0.45	0.1736	-0.11	-15.25

5.5----6.5	5.5	-0.11	0.0438	-0.13	-18.02
6.5----7.5	6.5	0.23	0.0910	0.05	6.93
7.5----8.5	7.5	0.57	0.2157	0.12	16.63
8.5----9.5	8.5	0.91	0.3186	0.10	13.86
9.5----10.5	9.5	1.24	0.3925	0.07	9.70
10.5--+∞	10.5	1.58	0.4429	0.05	6.93

Fig.1. Behavior of Avg RF and Avg GWLs frequencies for Normal distribution by using areas method in Zone –I, II, III, IV and V





Note: Average RF measured in Mille Meters and Average GWLs measured in Meters.

3. Conclusion

By Comparing Average RF and Average GWLs frequencies through Normal distribution by using areas method under consideration, for Average RF and Average GWLs frequencies follows Normal distribution clearly as shown in the above fig.1.

Declarations

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Competing Interests Statement

The author declares no competing financial, professional and personal interests.

Consent for publication

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