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Heat Transfer Analyses for Falkner-Skan Boundary Layer Flow (BLF) past a Static Wedge with respect to Velocity Slip Condition and Temperature-Dependent Thermal Conductivity

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ABSTRACT

This article is mainly studying the heat transfer for Falkner-Skan BLF passing a wedge, by considering N and A, they are: slip condition and variable thermal conductivity respectively. So, by using scaling group of transformation technique, the BLEs that are governing the problem has been transformed to a set of ODEs. Then, by using Runge–Kutta–Fehlberg method, the system is solved numerically to obtain that the heat transfer rate and the fluid temperature are decreasing whenever the A increase. Also, there is an increase of the N to increase the rate of the heat transfer and velocity as well. N is also decreasing both of the shear stress coefficient and temperature. Nevertheless, the shear stress coefficient and the velocity rate of the heat transfer respectively are increasing with m while the fluid temperature decreases with m, such that, m is the Falkner-Skan power law parameter. A compression is done between the results obtained at the end of this paper with previous published results.

Keywords: *Heat transfer, Falkner-skan boundary layer flow (BLF), Wedge, Velocity slip, Thermal conductivity.*

Highlights

(1) Introduce the problem of Falkner-Skan boundary layer flow over a static wedge by considering certain effects.

(2) Applying the scaling transformations to the problem as a similarity representation and then a numerical solution is done to indicate the effects of the influence parameters.

(3) Compare the present results with previous findings.

1. Introduction

Usually, the PDEs system is difficult to solve. So, an advanced transformation technique called similarity transformations are introduced to transform an original system of PDEs to a simplified ODE system. In studying a flow over a static wedge that is submerged in a viscous fluid, Falkner and Skan in (1931) reduce the boundary layer PDEs to a nonlinear third-order ODE by developing a similarity transformation. Diversity effects have been considered since then by many scholars, one of which developed a new numerical technique to transform the equation that governs the problem under study into a non-linear second-order BVP and uses the Lie-group shooting method on Blasius and Falkner–Skan equations to solve it (Liu and Chang 2008). Another study used the Adomian decomposition method to solve the momentum equation of the Falkner–Skan equation for accelerated flow and decelerated flow cases, such that, $m > 0, \beta > 0$ and $m < 0, \beta < 0$ where m is the power of length coordinate respectively, on a steady state BLF (Alizadeh et al. 2009). A discussion carried on the effect of suction and injection on tangential movement of a non-linear power-low stretching surface that governed by the laminar BLF and incompressible fluid (Afzal 2010). Also, based on a new approximate method called pseudo-spectral for the

Hermite functions to solve the third order nonlinear ordinary differential laminar boundary layer Falkner–Skan equations on the semi-infinite domain (Parand et al. 2011). Correspondingly, dual solutions are obtained to the BLF of a non-Newtonian power-law fluid over a wedge with suction or injection, which stretches towards or away from a variable free stream (Postelnicu and Pop 2011). In the same vein, it has been found that the slip BC have been frequently applied in the flow problems computationally numerical being examined to figure out its influence on the solution when a rotational body force field exists (Chen et al. 1981). On the other hand, other scholars such as, (Hayat et al. 2007), (Xiao et al. 2009), (Rahman and Eltayeb 2010) and (Li and An 2011), conducted studies on the flow of a Newtonian and non-Newtonian fluid with heat transfer, taking into account slip conditions. In addition, because of the direct relation between thermal conductivity and temperature (if the region of the temperature is large), some researchers such as, (Abel et al. 2009), (Ahmad et al. 2010), (Shang 2011) and (Mierzwiczak et al. 2011) used temperature-dependent thermal conductivity in their studies by considering the thermal conductivity as a function of the temperature. This study aims to consider the problem of Falkner-Skan BLF over a static wedge by taking into consideration the effect of the N and A parameters. A similarity representation of the problem is presented by applying the scaling transformations method where it has been solved numerically to indicate the effects of the influence parameters, viz-a-viz: N, A and m, where, m is Falkner–Skan power law parameter.

2. Mathematical Formalism of the Problem

Take into account the steady two-dimensional Falkner-Skan BLF past a static wedge as illustrated in Figure (1); where, the effects of N and A parameters are considered. Assume that the velocity of the free stream is $u_e = U_\infty x^m$ and the Cartesian coordinate system (x, y), where x and y are the coordinates measured along the surface of the wedge and normal to it (Yacob et al. 2011). Now, the system of PDEs is given as:

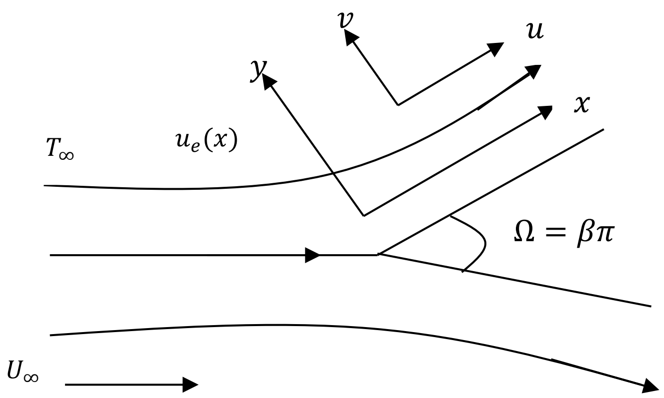

Fig.1. The physical model and its coordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + v\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left[k(T)\frac{\partial T}{\partial y}\right] \tag{3}$$

and their corresponding BCs as in Hayat et al. (2007) are:

$$y = 0, u = N_1(x)v\frac{\partial u}{\partial y}, v = 0, T = T_w \quad (4)$$

$$y \to \infty, u = u_e(x), T = T_\infty$$

Such that, u and v are the velocity components along the x and y axes, ρ is the fluid density, c_p is the specific heat, υ is the kinematic viscosity, N_1 is the slip parameter, T is the temperature, T_∞ is the free stream temperature and k is the thermal conductivity. The following relations for u, v, θ and k are introduced as:

$$k(T) = k_\infty[1 + c(T - T_\infty)], \qquad \text{(Abel et al., 2009)} \qquad (5)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$

Where, c and k_∞ are constants, ψ is the stream function and θ is the dimensionless temperature. Then, equations (1) and (3) transform as:

$$\psi_y\psi_{xy} - \psi_x\psi_{yy} = u_e\frac{\partial u_e}{\partial x} + \upsilon\psi_{yyy} \quad (6)$$

$$\psi_y\theta_x - \psi_x\theta_y = \alpha\frac{\partial}{\partial y}\left[(1 + A\theta)\frac{\partial \theta}{\partial y}\right] \quad (7)$$

And the BCs in (4) transforms to:

$$y = 0, \psi_y = N_1\upsilon\psi_{yy}, \psi_x = 0, \theta = 1 \quad (8)$$

$$y \to \infty, \psi_y = u_e(x), \theta = 0$$

Where, $A = c(T_w - T_\infty)$ is the thermal conductivity parameter and $\alpha = \frac{k_\infty}{\rho c_p}$ is the thermal diffusivity.

Equations (6)-(8) cannot be solved to get a closed-form solution; so transforming this system to an ordinary system using scaling transformations:

$$x^* = \lambda^{\varepsilon c_1}x, y^* = \lambda^{\varepsilon c_2}y, \psi^* = \lambda^{\varepsilon c_3}\psi, \theta^* = \lambda^{\varepsilon c_4}\theta \quad (9)$$

Where, the c's are constants (Mukhopadhyay et al., 2005).

Then, (6)-(8) stay invariant with the group of transformations in (9) if:

$$c_2 = \frac{1}{2}(1 - m)c_1, c_3 = \frac{1}{2}(1 + m)c_1, c_4 = 0 \quad (10)$$

and the characteristic equations become:

$$\frac{dx}{c_1 x} = \frac{dy}{\frac{1}{2}(1-m)c_1 y} = \frac{d\psi}{\frac{1}{2}(1+m)c_1\psi} = \frac{d\theta}{0} \quad (11)$$

Solving the equations above resulted that:

$$\eta = x^{\frac{1-m}{2}}y, \psi = x^{\frac{1-m}{2}}f(\eta), \theta = \theta(\eta) \quad (12)$$

Substituting from (12) into (6) and (7):

$$\nu f''' + \frac{m+1}{2} f f'' - m(f')^2 + mU_\infty^2 = 0 \quad (13)$$

$$\alpha(1 + A\theta)\theta'' + \frac{m+1}{2} f\theta' + \alpha A(\theta')^2 = 0 \quad (14)$$

Where, prime is the derivative with respect to η. Then, the BCs become:

$$f(0) = 0,\ f'(0) = \tilde{N}(x) f''(0),\ \theta(0) = 1 \quad (15)$$

$$f'(\infty) \to U_\infty,\ \theta(\infty) = 0\ ,$$

where $\tilde{N}(x) = \nu N_1(x) x^{(m+1)/2}$.

The following are dimensionless variables:

$$\tilde{\eta} = \sqrt{\frac{U_\infty(m+1)}{2\nu}}\eta,\ N_1(x) = \frac{\sqrt{2}\,N}{\sqrt{\nu U_\infty (m+1) x^{m+1}}},\ f = \sqrt{\frac{2\nu U_\infty}{(m+1)}} f(\tilde{\eta}),\ \theta(\eta) = \theta(\tilde{\eta}) \quad (16)$$

Where, N is the constant velocity slip parameter.

Using (16), (13) and (14) become:

$$f''' + f f'' + \frac{2m}{m+1}[1 - (f')^2] = 0 \quad (17)$$

$$(1 + A\theta)\theta'' + f\theta' + A(\theta')^2 = 0 \quad (18)$$

and the corresponding BCs (15) become:

$$f(0) = 0,\ f'(0) = N f''(0),\ \theta(0) = 1 \quad (19)$$

$$f'(\infty) = 1,\ \theta(\infty) = 0$$

3. Numerical Results and Discussions

Numerical results are obtained to study the effect of the various values of the parameters A, m and N on velocity, temperature, shear stress coefficient and rate of heat transfer.

For this, equations (17) and (18) together with their corresponding BCs in (19) have been solved numerically using Runge–Kutta–Fehlberg fourth fifth method. Velocity $f'(\eta)$ and temperature $\theta(\eta)$ findings are illustrated in Figures (2)-(7); while the coefficient of the skin friction $f''(0)$ and the rate at the wall heat transfer $\theta'(0)$ are presented in Table (1).

It is noticed from Table (1) that the increase of A decreases the rate of heat transfer. One also can see that the increase of m increases the wall velocity, the shear stress and the heat transfer rate at the wall.

Further, the wall velocity and the wall heat transfer rate increase while the skin friction coefficient decreases with the increase of the slip parameter N.

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Table 1. Values of velocity profiles, shear stress coefficient and rate of wall heat transfer for different values A, m, N

m	N	$f'(0)$	$f''(0)$	$-\theta'(0)$			
		$A = 0.5$	$A = 0.5$	$A = 0.5$	$A = 0.8$	$A = 1$	$A = 2$
0.2	0.1	0.07674	0.76741	0.43146	0.38974	0.36810	0.29831
	0.4	0.26423	0.66059	0.47328	0.42576	0.40114	0.32200
	0.7	0.39737	0.56767	0.50086	0.44952	0.42293	0.33759
	1	0.49300	0.49300	0.51978	0.46581	0.43788	0.34829
0.5	0.1	0.09721	0.97208	0.45428	0.40980	0.38672	0.31226
	0.4	0.31641	0.79102	0.49633	0.44591	0.41978	0.33577
	0.7	0.45775	0.65392	0.52179	0.46771	0.43980	0.35000
	1	0.55311	0.55311	0.53835	0.48200	0.45283	0.35926
0.8	0.1	0.10836	1.08361	0.46500	0.41920	0.39543	0.31875
	0.4	0.34292	0.85730	0.50686	0.45509	0.42826	0.34200
	0.7	0.48706	0.69581	0.53112	0.47589	0.44729	0.35548
	1	0.58147	0.58147	0.54648	0.48907	0.45935	0.36402
1.2	0.1	0.11737	1.17366	0.47292	0.42614	0.40186	0.32351
	0.4	0.36339	1.17366	0.51449	0.46173	0.43439	0.34650
	0.7	0.50909	0.72727	0.53777	0.48167	0.45262	0.35937
	1	0.50909	0.60242	0.55222	0.49406	0.46395	0.36736

The effect of m on the velocity and the temperature profiles are shown in Figures (2) and (3), respectively for $A = 1$ and $N = 1$.

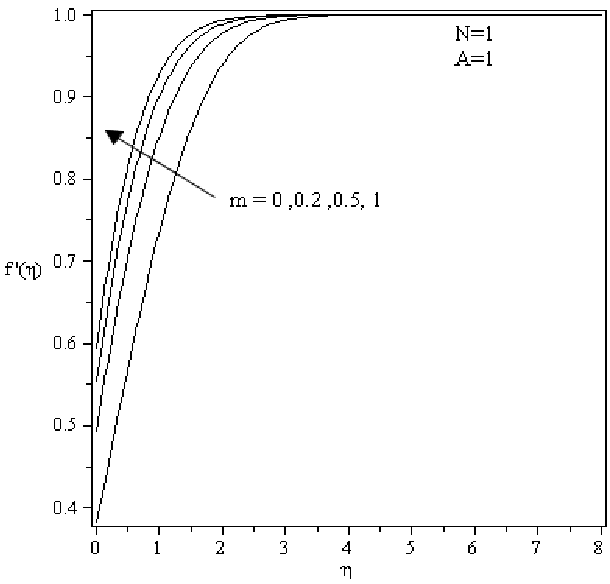

Fig.2. Effects of m on the velocity, when $A = 1$ and $N = 1$

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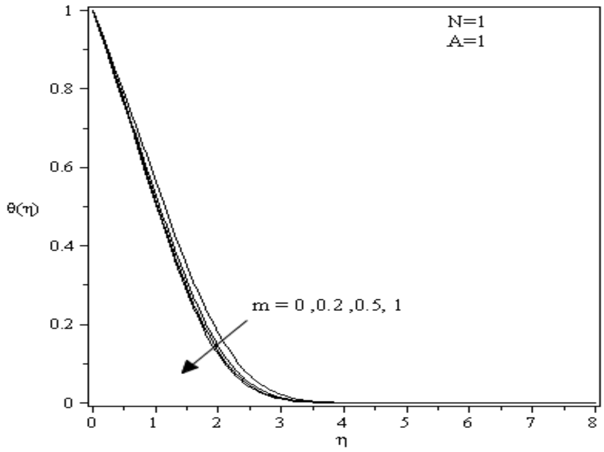

Fig.3. Effects of m on the temperature, when $A = 1$ and $N = 1$

Figure (2) shows that the velocity increases as m increases while Figure (3) the temperature profiles decreases with increasing values of m.

Figures (4) and (5) represents the effect of A on the velocity and temperature profiles with $m = 1$ and $N = 1$.

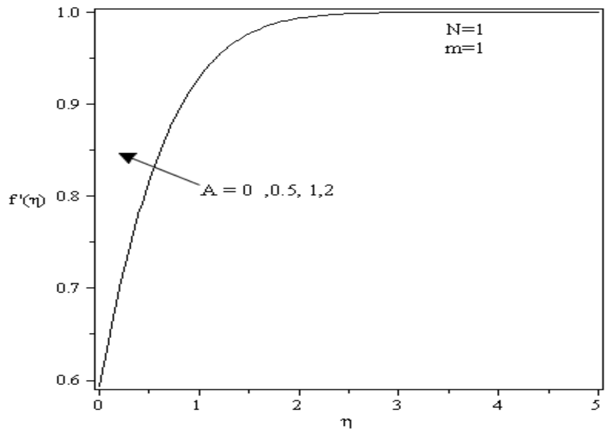

Fig.4. Effects of A on the velocity, when $m = 1$ and $N = 1$

It is clear that there is no change in the velocity profiles as in Figure (4) while from Figure (5) the temperature decreases when A increases.

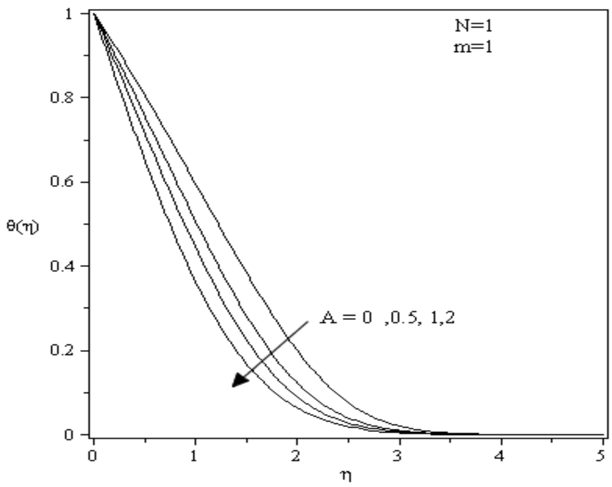

Fig.5. Effects of A on the temperature, when $m = 1$ and $N = 1$

Figures (6) and (7) depict the effects of N on the velocity and temperature profiles, respectively, with $m = 1$ and $A = 1$. One can see clearly that the velocity increases with the increase of N, while the temperature behaviour is just the opposite.

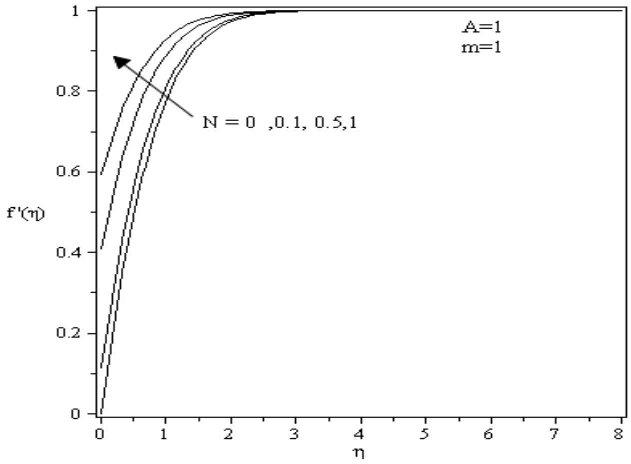

Fig.6. Effects of N on the velocity, when $A = 1$ and $m = 1$

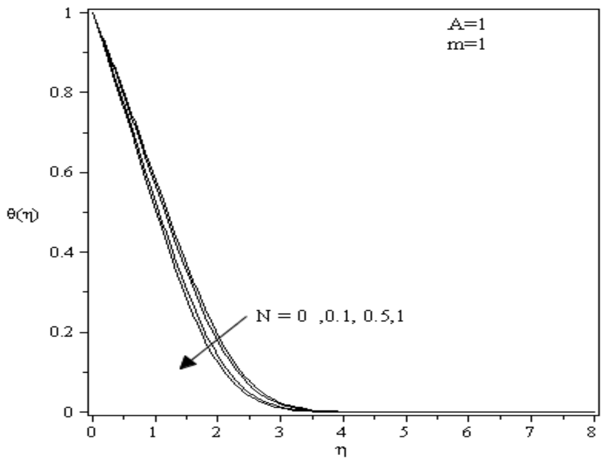

Fig.7. Effects of N on the temperature, when $A = 1$ and $m = 1$

Never the less, the results obtained have been compared with previous published findings by (Watanabe 1990), (Yih 1998) and (Yacob et al. 2011) when $A = 0$, $m = 0$ and $N = 0$. It can be seen that there is an excellent agreement between the present and previous findings as it is shown in Table (2).

Table 2. The values of $f''(0)$ for various values of m when $A = 0$ and $N = 0$

m	Watanabe (1990)	Yih (1998)	Eycob (2011)	Present Results
0	0.46960	0.649600	0.4696	**0.46960007**
1/11	0.65498	0.654979	0.6550	**0.65499372**
0.2	0.80213	0.802125	0.8021	**0.80212560**
1/3	0.92765	0.927653	0.9277	**0.92768004**
0.5			1.0389	**1.03890348**
1		**1.232588**	**1.2326**	**1.23258764**

4. Conclusion

This paper presented a study on heat transfer analysis for Falkner-Skan BLF past a wedge with respect to the slip condition considering temperature-Dependent thermal conductivity.

The system of PDEs was converted to a non-linear system of ODEs using scaling transformation analysis and then solved numerically by Runge–Kutta– Fehlberg method. It is noticed from the numerical results that the velocity, rate of heat transfer at the wall and skin friction coefficient increase with m, but the fluid temperature decrease.

The increase of N yields to an increase of the velocity and the wall heat transfer rate, while it decreases the temperature and shear stress. The fluid temperature and wall heat transfer rate decrease with the A.

Declarations

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Competing Interests Statement

The authors declare no competing financial, professional and personal interests.

Consent to participate

Not Applicable

Consent for publication

We declare that we consented for the publication of this research work.

Availability of data and material

Authors are willing to share data and material according to the relevant needs.

Author's contribution

All authors participated in overseeing laboratory work, data analysis, and manuscript writing and review.

References

1) Abel, M.S., Siddheshwar, P.G., Mahesha (2009) 'Effects of thermal buoyancy and variable thermal conductivity on the MHD flow and heat transfer in a power-law fluid past a vertical stretching sheet in the presence of a non-uniform heat source', International Journal of Non-Linear Mechanics, 44, 1-12.

2) Ahmad, N., Siddiqui, Z.U., .Mishra, M.K (2010) 'Boundary layer flow and heat transfer past a stretching plate with variable thermal conductivity', International Journal of Non-Linear Mechanics, 45, 306–309.

3) Afzal, N. (2010) 'Falkner–Skan equation for flow past a stretching surface with suction or blowing, Analytical solutions', Applied Mathematics and Computation, 217, 2724-2736.

4) Alizadeh, E., Farhadi, M., Sedighi, K., Ebrahimi-Kebria, H.R., Ghafourian, A., (2009) ‘Solution of the Falkner–Skan equation for wedge by Adomian Decomposition Method’, Communications in Nonlinear Science and Numerical Simulation, 14, 724–733.

5) Sin-Sung Chen, Chuen-Yen Chow, Mahinder S. Uberoi, (1981) "Effect of Slip boundary condition on flow computation in the presence of rational body forces", An International Journal of Computers and Fluids, Volume 9, Issue 4, 389-393.

6) Falkner, V.M., Skan, S.W., (1931) “Some Approximate Solutions of the Boundary-Layer Equations”, Philos. Mag. 12, 865–896

7) Hayat, T., Khan, M., Ayub, M., (2007) “The effect of the slip condition on flows of an Oldroyd 6-constant fluid”, Journal of Computational and Applied Mathematics 202, 402 – 413.

8) Liu, C.S., Chang, J.R., (2008) “The Lie-group shooting method for multiple-solutions of Falkner–Skan equation under suction–injection conditions”, International Journal of Non-Linear Mechanics 43, 844 – 851.

9) Li, Y., An, R., (2011) “Two-level pressure projection finite element methods for Navier–Stokes equations with nonlinear slip boundary conditions”, Applied Numerical Mathematics.

10) Mierzwiczak, M., Kołodziej, J.A., (2011) “The determination temperature-dependent thermal conductivity as inverse steady heat conduction problem”, International Journal of Heat and Mass Transfer 54, 790–796.

11) Mukhopadhyay, S., Layek, G.C., Samad, S.A., (2005) “Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity”, International Journal of Heat Mass Transfer, 48(2), 4460–4466.

12) Parand, K., Rezaei, A.R., Ghaderi, S.M., (2011) “An approximate solution of the MHD Falkner–Skan flow by Hermite functions pseudospectral method”, Commun Nonlinear Sci. Numer Simulat 16, 274–283.

13) Postelnicu, A., Pop, I., (2011) “Falkner–Skan boundary layer flow of a power-law fluid past a stretching wedge”, Applied Mathematics and Computation 217, 4359–4368.

14) Rahman, M. M., Eltayeb, I. A., (2010) “Convective slip flow of rarefied fluids over a wedge with thermal jump and variable transport properties”, International Journal of Thermal Sciences, 1-12.

15) Shang, D., (2011)“Theory of Heat Transfer with Forced Convection Film Flows, Part 2”, Pages 119-147,.

16) Watanabe, T., (1990) “Thermal boundary layers over a wedge with uniform suction or injection in forced flow”, Acta Mech. 83, 119-126.

17) Xiao, Y., Xin, Z., Wu, J., (2009) “Vanishing viscosity limit for the 3D magnetohydrodynamic system with a slip boundary condition”, Journal of Functional Analysis 257, 3375–3394.

18) Yih, K.A., (1998) “Uniform suction/blowing effect on forced convection about a wedge: uniform heat flux”, Acta Mech. 128, 173-181.

19) Yacob, N.A., Ishak, A., Pop, I., (2011) “Falkner- Skan problem for a static or moving wedge in nanofluids”, International Journal of Thermal Sciences 50, 133-139.